PERIOD–I

TOPIC

3 Constructions

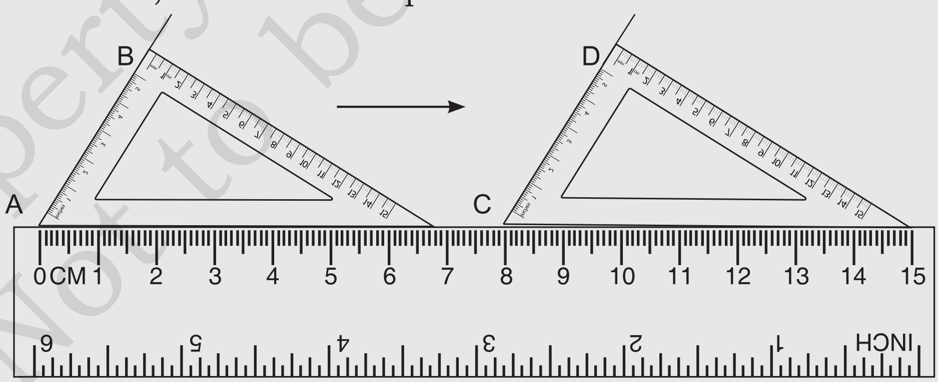
3.1. CONSTRUCTION WITHOUT MEASUREMENT

ACTIVITY 1

Aim: To draw parallel and perpendicular lines using a set square.

Materials Required: Set square, ruler, pencil.

- Divide the class in pairs.
- Instruction them to use a set square and ruler to draw parallel lines.
- Keep the set square on the ruler and draw $\overrightarrow{AB}$, now move the set square along the ruler as shown in figure.
- Draw $\overrightarrow{CD}$, observe $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$

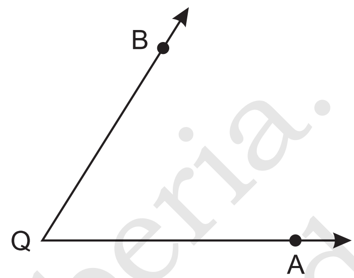


3.2. BISECTOR OF AN ANGLE

Bisecting an angle means drawing a ray in the interior of an angle such the angle is divided into two equal angles.

Construction of the Bisector of a Given Angle using a Pair of Compasses

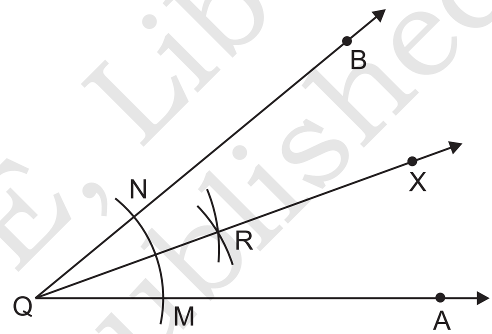
An angle ∠*AQB* is given. Let us draw a ray *OX*, bisecting ∠*AQB*.



Steps of Construction

Step 1: With Q as centre and any radius, draw an arc, cutting $\overrightarrow{QA}$ and $\overrightarrow{QB}$ at *M* and *N* respectively.

Step 2: With *M* as centre and radius more than half of *MN*, draw an arc. With centre at *N* and the same radius as before, draw another arc, cutting the previous drawn arc at a point *R*.



Step 3: Join *QR* and produce it to any point *X*.

Then, ray $\overrightarrow{QX}$ bisects ∠*AQB* into two equal angles, namely, ∠AQX, and ∠*BQX*.

3.3. CONSTRUCTION OF 75°, 105°, 135° AND 150°

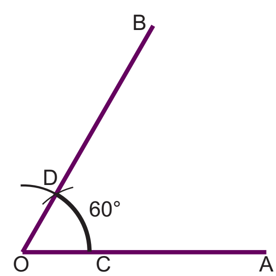
(a) Let us first revise the construction of 30°, 45°, 60° and 90°.

(i) To construct an angle of 60°

Steps of construction

1. Draw any straight line *OA*.
2. With *O* as centre and any (suitable) radius, draw an arc to meet *OA* at *C*.
3. With *C* as centre and same radius as in step 2, draw an arc to meet the previous arc at *D*.
4. Join *O, D* and produce it to *B*.

   Then ∠*AOB* = 60°.



(ii) To construct an angle of 30°

We observe that $\frac{0° + 60°}{2}$ = 30°, therefore, if ∠*AOB* = 60° and *OE* is the bisector of ∠*AOB,* then ∠*AOB* = 30°.

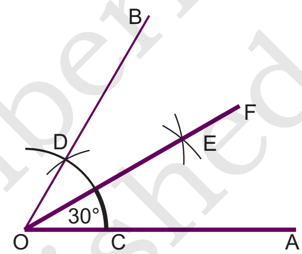
Steps of construction

1. Construct ∠*AOB* = 60°

2. With *C* as centre and any suitable radius (greater than half of *CD*), draw an arc. Also, with *D* as centre and same radius, draw another arc to meet the previous arc at *E*.

3. Join *O, E* and produce it to *F*. Then *OF* bisects ∠*AOB*.

Therefore, ∠*AOF* = 30°



(iii) To construct an angle of 90°

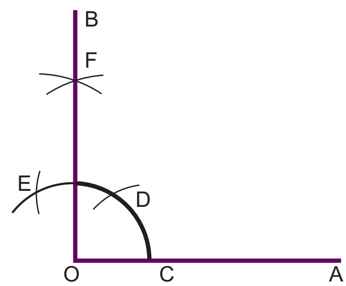
Steps of construction

1. Draw any straight line *OA*.

2. With *O* as centre and any suitable radius, draw an arc to meet *OA* at *C*.

3. With *C* as centre and same radius, (as in step 2) draw an arc to meet the previous arc at *D*. With *D* as centre and same radius, draw another arc to meet the first arc at *E*.

4. With *D* and *E* as centres draw two arcs of equal radius (greater than half of *CD*), cutting each other at *F*.



5. Join *O, F* and produce it to *B*. Then ∠*AOB* = 90°

Note that: $\frac{60° + 120°}{2} = \frac{180°}{2} = 90°$

Here ∠ *AOD* = 60°,

$$\angle AOE = 120° \text{ so that}$$
$$\angle DOE = 120° - 60° = 60°$$
$$\angle DOF = \frac{1}{2} \times 60° = 30°,$$

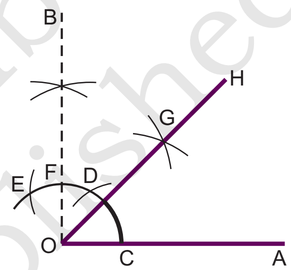
because *OF* is the bisector of ∠*DOE*.

$$\angle AOF = \angle AOD + \angle DOF$$
$$= 60° + 30° = 90°$$

(iv) To construct an angle of 45°

We observe that $\frac{0°+90°}{2} = 45°$, therefore, if $\angle AOB = 90°$ and OG is the bisector of $\angle AOB$, then $\angle AOG = 45°$

Steps of construction



1. Construct $\angle AOB = 90°$

2. With C as centre and any suitable radius greater than $\frac{1}{2}CF$, draw an arc. Also, with F as centre and same radius, draw another arc to meet the previous arc at G.

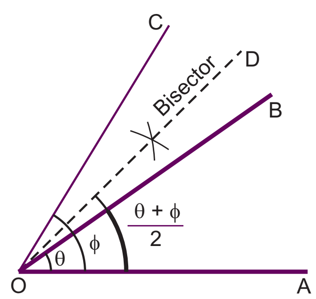
Note that: $\frac{0°+90°}{2} = \frac{90°}{2} = 45°$

Here OH is the bisector of $\angle AOB$,

3. Join O, G and produce it to H.
Then $\angle AOH = 45°$

(b) Now, let us construct angles of 75°, 105°, 135°, and 150°.

We observe that if $\angle AOB = \theta$ and $AOC = \phi$, then $\angle BOC = \angle AOC - \angle AOB = \phi - \theta$. If OD bisects $\angle BOC$, then

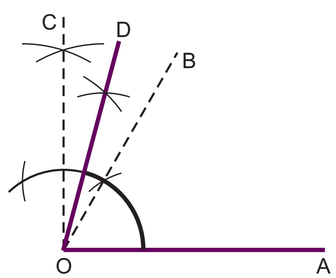


$$\angle BOD = \frac{1}{2}\angle BOC = \frac{\phi-\theta}{2} \text{ so that}$$
$$\angle AOD = \angle AOB + \angle BOD$$
$$= \theta + \frac{\phi-\theta}{2} = \frac{\theta+\phi}{2}.$$

(i) To construct an angle of 75°

Note that: $\frac{60°+90°}{2} = \frac{150°}{2} = 75°$

Steps of construction



1. Construct $\angle AOB = 60°$

2. Construct $\angle AOC = 90°$

then $\angle BOC = 90° - 60° = 30°$

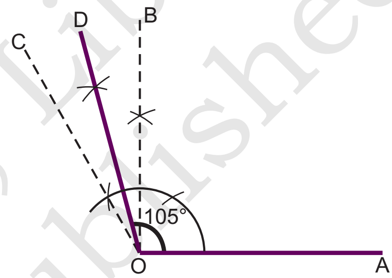
3. Let OD be the bisector of $\angle BOC$, then $\angle BOD = \frac{1}{2} \times 30° = 15°$, so that

$$\angle AOD = \angle AOB + \angle BOD$$
$$= 60° + 15° = 75°$$

(ii) To construct an angle of 105°

Note that: $\frac{90° + 120°}{2} = \frac{210°}{2} = 105°$

Steps of construction



1. Construct $\angle AOB = 90°$

2. Construct $\angle AOC = 120°$
then $\angle BOC = 120° - 90° = 30°$

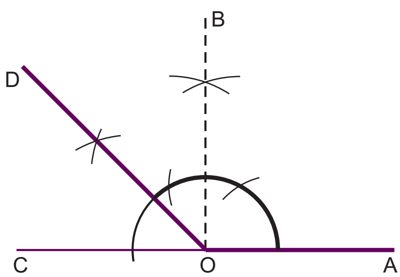
3. Let OD be the bisector of $\angle BOC$
then $\angle BOD = \frac{1}{2} \times 30° = 15°$, so that

$$\angle AOD = \angle AOB + \angle BOD$$
$$= 90° + 15° = 105°$$

(iii) To construct an angle of 135°

Note that: $\frac{90° + 180°}{2} = \frac{270°}{2} = 135°$

Steps of construction



1. Construct $\angle AOB = 90°$

2. Produce AO to C, then
$\angle AOC = 180°$
and $\angle BOC = 180° - 90° = 90°$

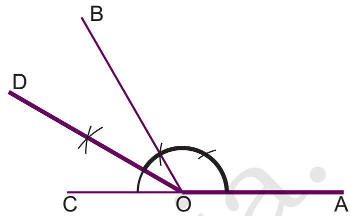
3. Let OD be the bisector of $\angle BOC$
then $\angle BOD = \frac{1}{2} \times 90° = 45°$, so that

$$\angle AOD = \angle AOB + \angle BOD$$
$$= 90° + 45° = 135°$$

(iv) To construct an angle of 150°

Note that: $\frac{120° + 180°}{2} = \frac{300°}{2} = 150°$

Steps of construction



1. Construct $\angle AOB = 120°$

2. Produce AO to C, then

$\angle AOC = 180°$ and

$\angle BOC = 180° - 120° = 60°$

3. Let OD be the bisector of $\angle BOC$, then $\angle BOD = \frac{1}{2} \times 60° = 30°$,

so that $\angle AOD = \angle AOB + \angle BOD$

$= 120° + 30° = 150°.$

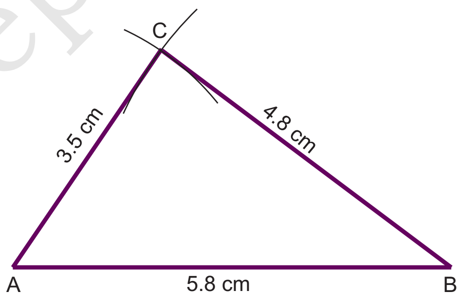
3.4. CONSTRUCTION OF TRIANGLES AND QUADRILATERALS

(a) Construction of Triangles

(i) To construct a triangle when the lengths of three sides are given

Example 1. *Construct a triangle ABC such that AB = 5.8 cm, BC = 4.8 cm and AC = 3.5 cm.*

Steps of construction



1. Draw line segment $AB = 5.8$ cm.

2. With B as centre and radius $= BC = 4.8$ cm, draw an arc,

3. With A as centre and radius $= AC = 3.5$ cm, draw an arc to cut the arc of step 2 at C.

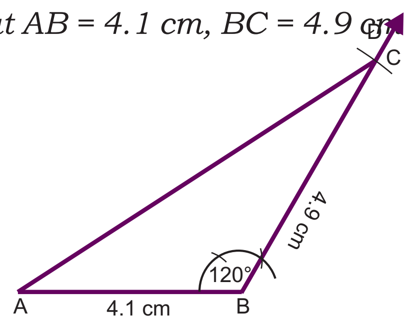
4. Join AC and BC.

Then ABC is the required triangle.

(ii) To construct a triangle when two sides and the included angle are given

Example 2. *Construct a triangle ABC such that AB = 4.1 cm, BC = 4.9 cm and ∠B = 120°.*

Steps of construction



1. Draw line segment $AB = 4.1$ cm.

2. At B, construct $\angle ABD = 120°$

3. From BD, cut off $BC = 4.9$ cm

4. Join AC.

Then ABC is the required triangle.

(iii) To construct a triangle when one side and two angles are given

When two angles say $\angle A$ and $\angle B$ of a triangle ABC are given, then since $\angle A + \angle B + \angle C = 180°$, the third angle $\angle C = 180° - \angle A - \angle B$ also becomes known.

Example 3. *Construct a triangle ABC such that BC = 7.5 cm, $\angle B = 45°$ and $\angle C = 60°$*

Steps of construction

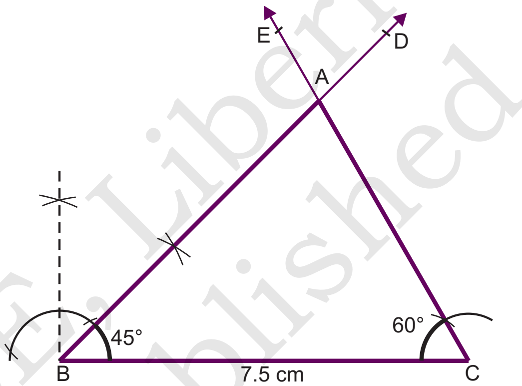
1. Draw line segment $BC = 7.5$ cm

2. At B, construct $\angle CBD = 45°$

3. At C, construct $\angle BCE = 60°$

4. Let BD and CE intersect at A.

Then ABC is the required triangle.



Example 4. *Construct an isosceles triangle ABC having base BC = 6.8 cm and $\angle B = 30°$.*

Since the base angles of an isosceles triangle are equal, therefore $\angle B = \angle C = 30°$

Steps of construction

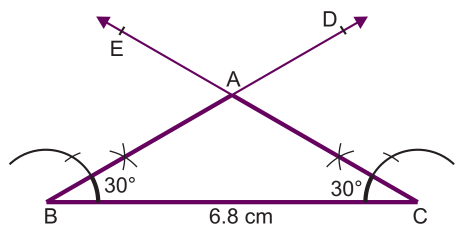
1. Draw line segment $BC = 6.8$ cm

2. At B, construct $\angle CBD = 30°$

3. At C, construct $\angle BCE = 30°$

4. Let BD and CE intersect at A.

Then ABC is the required isosceles triangle.



(iv) To construct a quadrilateral when three sides and two diagonals are given

Example 5. *Construct a quadrilateral ABCD in which AB = 5.5 cm, AD = 4.4 cm, CD = 6.5 cm , AC = 6.5 cm and BD = 7.1 cm.*

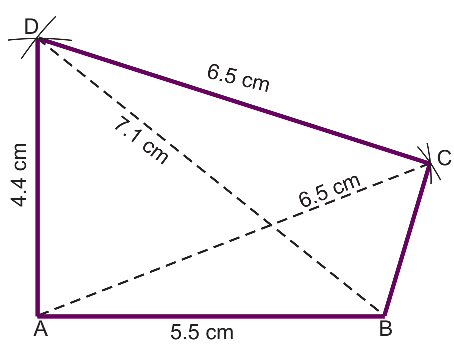
Steps of construction

1. Construct triangle ABD

2. Construct triangle ACD.

3. Join BC.

Then $ABCD$ is the required quadrilateral.



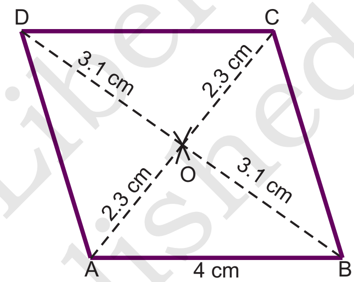
(v) To construct a parallelogram whose one side and both diagonals are given

To construct parallelograms, we use the following facts:

1. Opposite sides of a parallelogram are equal.

2. Diagonals bisect each other.

Example 6. *Construct a parallelogram ABCD given that AB = 4 cm, AC = 4.6 cm and BD = 6.2 cm.*



Steps of construction

1. Construct triangle OAB with

$AB = 4$ cm

$AO = \frac{1}{2}\ AC = \frac{1}{2} \times 4.6 = 2.3$ cm and

$BO = \frac{1}{2} BD = \frac{1}{2} \times 6.2 = 3.1$ cm.

2. Produce AO to C such that $OC = OA$

3. Produce BO to D such that $OD = OB$.

4. Join BC, CD, AD.

Then, $ABCD$ is the required parallelogram.

3.5. CONSTRUCTING LOCI

Locus is the set of all points in a plane which satisfy one or more geometrical conditions.

Alternatively, locus of a moving point is the path traced by the point under a given set of geometrical conditions.

Important facts about locus

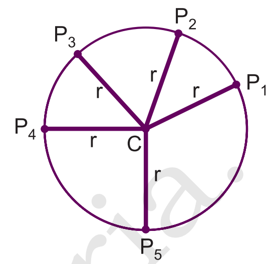
1. Every point which satisfies the given geometrical condition(*s*) lies on the locus.

2. Every point which lies on the locus satisfies the given geometrical condition(*s*).

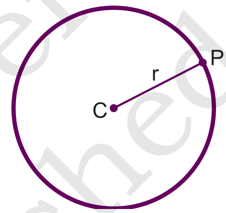
Thus, a point P lies on locus $\Leftrightarrow$ P satisfies the given geometrical condition(*s*).

Example 7. *Let $\{P_1, P_2, P_3, \ldots\}$ be the set of points in a plane such that their distance from a fixed point C in the plane is a positive constant r. Then, $CP_1 = CP_2 = CP_3 = \ldots. = r$.*

Solution. We observe that the curve through all these point is a circle with the fixed point C as centre and the constant distance r as radius.



Conversely, if P is any point on the circle then its distance from the centre C is the radius r, i.e, $CP = r$ for all points P on the circle.



Thus, *a circle is the set of all points in a plane (i.e. locus) which are at a constant distance from a fixed point in the plane. The fixed point is the centre and the constant distance is the radius of the circle.*

EXERCISE

1. Construct a quadrilateral $ABCD$ such that AB = 4.2 cm, BC = 3.7 cm, CD = 4.3 cm, AD = 3.1 cm and $\angle A = 60°$.
2. Construct a quadrilateral $ABCD$ in which AB = 4.4 cm, BC = 4 cm, CD = 6.4 cm, AD = 2.8 cm and BD = 6.6 cm.
3. Draw a parallelogram $ABCD$ in which AC = 6.8 cm, BD = 5 cm and an angle between them is 60°.
4. Construct a triangle ABC in which BC = 8 cm, AB = 5 cm and $\angle ABC = 60°$. Mark a point P which is equidistant from AB, BC and also from B and C.
5. Draw a line segment of length 6.6 cm. Bisect it and measure the length of each part.
6. Construct a ΔABC in which BC = 3.6 cm, $AB + AC$ = 4.8 cm and $\angle B = 60°$.